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Math 2010, Winter 2005, Quiz 11

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Consider the curve $y = \frac{x}{x^2+3}$.

1. Graph the curve, showing where it is increasing, where it is decreasing, where it is concave up, where it is concave down, and its limits as $x \to \pm\infty$.
2. What are the maximum and minimum values of y?

1.

$\lim_{x \to \pm\infty} y = 0$ so $y=0$ is a horizontal asymptote at "both ends".

Precisely $y \to 0^+$ as $x \to \infty$ and $y \to 0^-$ as $x \to -\infty$

No vertical asymptotes since x^2+3 is never 0.

$$y' = \frac{1(x^2+3) - x(2x)}{(x^2+3)^2} = \frac{3-x^2}{(x^2+3)^2}$$

$$y'' = \frac{-2x(x^2+3)^2 - (3-x^2)(2(x^2+3)(2x))}{(x^2+3)^4}$$

$$= \frac{-2x(x^2+3) - 4x(3-x^2)}{(x^2+3)^3}$$

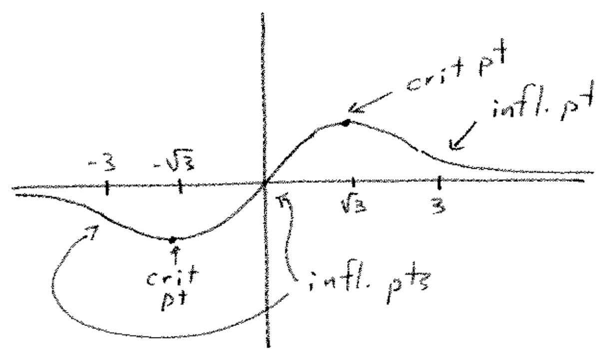
$$= \frac{-2x[x^2+3+2(3-x^2)]}{(x^2+3)^3}$$

$$= \frac{-2x[9-x^2]}{(x^2+3)^3} = \frac{2x[x^2-9]}{(x^2+3)^3}$$

$y' = 0$ iff $x = \pm\sqrt{3}$

$y'' = 0$ iff $x = 0, \pm 3$

		-3		$-\sqrt{3}$		0		$\sqrt{3}$		3	
y'	$-$ ↘			0		$+$ ↗		0		$-$ ↘	
y''	$-$ ⌢	0	$+$ ⌣			0	$-$ ⌢			0	$+$ ⌣



2. Max occurs at $x = \sqrt{3}$ so

$$\boxed{max = \frac{\sqrt{3}}{3+3} = \frac{\sqrt{3}}{6}}$$

min occurs at $x = -\sqrt{3}$ so

$$\boxed{min = \frac{-\sqrt{3}}{6}}$$