

Sample Test 4 Solutions

1. (a) $\sqrt{3x^3 - x}$

(b) $x^2 \sqrt{3-x^2} (2x)$

(c) $-x\sqrt{3-x}$

(d) 0 (deriv. of a constant)

(e) $1+x+x^2+x^3$

(f) $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$

(g) $e^x + C$

(h) $-\cos x + C$

(i) $\sin x + C$

(j) $\ln|\sec x| + C$

(k) $\tan x + C$

(l) $\sec x + C$

(m) $\ln|x| + C$

(n) $\frac{1}{1-3} x^{1-3} = -\frac{1}{2} x^{-2} + C$

(o) $\frac{1}{3/2} x^{3/2} + C = \frac{2}{3} x^{3/2} + C$

(p) $\int x^{3/2} + x^{1/2} dx = \frac{2}{5} x^{5/2} + \frac{2}{3} x^{3/2} + C$

(q) $u = \sqrt{x} \quad = \int \sin(u) \cdot 2du$
 $du = \frac{dx}{2\sqrt{x}} \quad = -2 \cos(\sqrt{x}) + C$

(r) $\frac{1}{2} \left[\frac{1}{4} (2x-3)^4 + \frac{1}{3} (2x-3)^3 + \frac{1}{2} (2x-3)^2 \right] + C$

(s) $\frac{1}{3} (-\cos(3x-2)) + C$

(t) $14 e^{(x+5)/7} + C$

(u) $e^{\sin x} + C$

(v) $\int_0^1 3+x dx = 3 + \frac{1}{2}$

(w) $= \int_{-1}^4 2 dx = 2 \cdot 8 = 16$

2. $-4 \leq f(x) \leq -2$ on $[0, 2]$

gives $\int_0^2 -4 dx \leq \int_0^2 f(x) dx \leq \int_0^2 -2 dx$

which says

$$-8 \leq \int_0^2 f(x) dx \leq -4$$

3. subdivide $[1, 4]$ into $[1, 2]$, $[2, 3]$, $[3, 4]$

pick sample points: I'll choose $\frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ (mid points). Then

$$\int_1^4 \frac{1}{x} dx \approx \frac{1}{3/2} \cdot 1 + \frac{1}{5/2} \cdot 1 + \frac{1}{7/2} \cdot 1$$

(widths are all 1)

$$= \frac{2}{3} + \frac{2}{5} + \frac{2}{7}$$

$$= \frac{142}{105}$$