

Sample test 4 solutions page 2

4. (a)  $\int_1^5 f(x) dx = \int_0^5 f(x) dx - \int_0^1 f(x) dx = 12 - 3 = \boxed{9}$

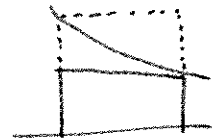
(b)  $\int_1^4 78f(x) dx = 78 \int_1^4 f(x) dx = 78 \left( \int_1^5 f(x) dx - \int_4^5 f(x) dx \right)$  but this is no help since we do not know  $\int_4^5$  yet, so  
 $= 78 \left( \int_0^4 f(x) dx - \int_0^1 f(x) dx \right) = 78(4-3) = \boxed{78}$

5. Given  $\int_2^5 4f(x) - 8g(x) dx = 4$  we get

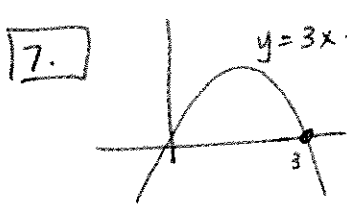
$4 = 4 \int_2^5 f(x) dx - 8 \int_2^5 g(x) dx = 4(7) - 8 \int_2^5 g(x) dx$ , so

$8 \int_2^5 g(x) dx = 28 - 4 = 24$ . Hence  $\int_2^5 g(x) dx = \frac{24}{8} = \boxed{3}$ .

6.  $f$  decreasing  $\Rightarrow$  left endpoint overestimates and right endpoint underestimates.

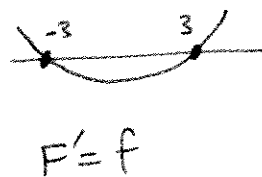
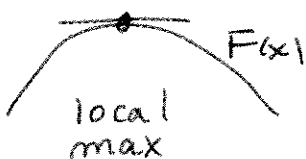


Intervals:	[0,2]	[2,3]	[3,5]	[5,6]
Underestimate =	$5 \cdot 2$	$+ 4 \cdot 1$	$+ 1 \cdot 2$	$+ (-2) \cdot 1 = 10 + 4 + 2 - 2 = \boxed{14}$
Overestimate $\Rightarrow$	$8 \cdot 2$	$+ 5 \cdot 1$	$+ 4 \cdot 2$	$+ 1 \cdot 1 = 16 + 5 + 8 + 1 = \boxed{30}$



Area =  $\int_0^3 3x - x^2 dx = \left. \frac{3}{2}x^2 - \frac{1}{3}x^3 \right|_0^3$   
 $= \frac{3}{2}(3^2) - \frac{1}{3}(3^3) = 3^2 \left( \frac{3}{2} - 1 \right) = \frac{3^2}{2} = \boxed{\frac{9}{4}}$

8.  $F(x) = \int_0^x f(t) dt$  has a local maximum if  $F' = 0$  at that point,  $F' > 0$  before, and  $F' < 0$  after. Since  $F' = f$ , this happens at  $x = -3$  and nowhere else.



$\boxed{\text{At } x = -3.}$