

**TATE COHOMOLOGY OF THE ANTI-INVOLUTION OF THE STEENROD
ALGEBRA
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ABSTRACT. A MAGMA calculation

1. THE RESULT

This is a calculation by brute force of the Tate cohomology of the cyclic group of order 2 with coefficients in the mod 2 Steenrod algebra with the canonical involution as the C_2 action; the generators are complete through degree 225 and the relations are complete through degree 200.

There are generators $a_{2^n-2} = \xi_{n-1}\chi(\xi_{n-1})$ and a_{2^n+1} as found by Crossley and Whitehouse [CW]. (They called them (more sensibly) a_{n-1} and d_n , respectively.) In addition we find generators in degrees

$$49 : = 32 + 17$$

$$81 : = 64 + 17$$

$$97 : = 64 + 33$$

$$145 : = 128 + 17$$

$$161 : = 128 + 33$$

$$193 : = 128 + 65$$

$$225 : = 128 + 97$$

Note that the generators of the Tate cohomology do not suffice to generate the fixed points. The first counterexample to this lies in degree 22, where the norm $(1 + \chi)(\xi_3\xi_4)$ is fixed but cannot be written in terms of lifts of the generators of the Tate cohomology.

The relations are calculated by computing a Gröbner basis for the ideal generated by the relations in degrees up to 200. It appears that the generators other than those in degrees $2^n - 2$ all satisfy $x^3 = 0$. This is verified for $a_1, a_9, a_{17}, a_{33}, a_{49}, a_{65}, a_{81},$ and a_{97} .

There are also suggestive patterns in the powers of the $a_{2^n-2} = \xi_{n-1}\chi(\xi_{n-1})$. We have $a_6^3 = a_9^2$ and $a_{14}^7 = a_{49}^2$, and we conjecture that $a_{30}^{15} = a_{225}^2$, and more generally, that $a_{2^{n+1}-2}^{2^n-1} = a_{(2^n-1)^2}^2$, which requires that there be generators $a_{225}, a_{(31)^2}, \dots$

Ring generators through degree 225:

$$a_1 := \xi_1$$

$$a_6 := \xi_1^3\xi_2 + \xi_2^2$$

$$a_9 := \xi_1^2\xi_3 + \xi_2^3$$

$$a_{14} := \xi_1^7\xi_3 + \xi_1^4\xi_2\xi_3 + \xi_1\xi_2^2\xi_3 + \xi_3^2$$

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Sarah Whitehouse and Martin Crossley caught a silly error in the first version of this calculation: I was only calculating the relations in the Steenrod algebra not modulo norms.

- (5) $a_1 a_{17} + a_9^2$
- (6) $a_1 a_{30} + a_{14} a_{17}$
- (7) $a_1 a_{33} + a_{17}^2$
- (8) $a_1 a_{49}$
- (9) $a_1 a_{62} + a_{14} a_{49} + a_{30} a_{33}$
- (10) $a_1 a_{65} + a_{33}^2$
- (11) $a_1 a_{81}$
- (12) $a_1 a_{97} + a_{49}^2$
- (13) $a_1 a_{126} + a_{30} a_{97} + a_{62} a_{65}$
- (14) $a_1 a_{129} + a_{65}^2$
- (15) $a_1 a_{145}$
- (16) $a_1 a_{161} + a_{81}^2$
- (17) $a_1 a_{193} + a_{97}^2$
- (18) $a_6^3 + a_9^2$
- (19) $a_6^2 a_9$
- (20) $a_6^2 a_{14} + a_9 a_{17}$
- (21) $a_6^2 a_{30} + a_9 a_{33}$
- (22) $a_6^2 a_{62} + a_9 a_{65}$
- (23) $a_6^2 a_{126} + a_9 a_{129}$
- (24) $a_6 a_9^2$
- (25) $a_6 a_9 a_{30} + a_{14}^2 a_{17}$
- (26) $a_6 a_9 a_{62} + a_{14}^2 a_{49} + a_{17} a_{30}^2$
- (27) $a_6 a_9 a_{126} + a_{17} a_{62}^2 + a_{30}^2 a_{81}$
- (28) $a_6 a_{14}^2 + a_{17}^2$
- (29) $a_6 a_{14} a_{30} + a_{17} a_{33}$
- (30) $a_6 a_{14} a_{62} + a_{17} a_{65}$

- (31) $a_6 a_{14} a_{126} + a_{14}^4 a_{30}^3 + a_{17} a_{129} + a_{65} a_{81}$
- (32) $a_6 a_{17} + a_9 a_{14}$
- (33) $a_6 a_{30}^2 + a_{33}^2$
- (34) $a_6 a_{30} a_{62} + a_{33} a_{65} + a_{49}^2$
- (35) $a_6 a_{30} a_{126} + a_{14}^5 a_{30} a_{62} + a_{33} a_{129} + a_{81}^2$
- (36) $a_6 a_{33} + a_9 a_{30}$
- (37) $a_6 a_{49}$
- (38) $a_6 a_{62}^2 + a_{49} a_{81} + a_{65}^2$
- (39) $a_6 a_{62} a_{126} + a_{14}^3 a_{30}^3 a_{62} + a_{49} a_{145} + a_{65} a_{129} + a_{97}^2$
- (40) $a_6 a_{65} + a_9 a_{62}$
- (41) $a_6 a_{81}$
- (42) $a_6 a_{97}$
- (43) $a_6 a_{129} + a_9 a_{126}$
- (44) $a_6 a_{145} + a_{14} a_{17} a_{30}^4$
- (45) $a_6 a_{161} + a_{17} a_{30}^5$
- (46) $a_6 a_{193} + a_{17} a_{30}^4 a_{62}$
- (47) a_9^3
- (48) $a_9^2 a_{17}$
- (49) $a_9^2 a_{30} + a_{14} a_{17}^2$
- (50) $a_9^2 a_{33}$
- (51) $a_9^2 a_{62} + a_{17} a_{30} a_{33}$
- (52) $a_9^2 a_{65}$
- (53) $a_9^2 a_{126} + a_{17} a_{62} a_{65} + a_{30} a_{49} a_{65}$
- (54) $a_9^2 a_{129}$
- (55) $a_9^2 a_{193}$
- (56) $a_9 a_{14}^2$

- (57) $a_9 a_{14} a_{17}$
- (58) $a_9 a_{14} a_{30} a_{62} + a_{17} a_{33} a_{65}$
- (59) $a_9 a_{14} a_{30} a_{126} + a_{17} a_{33} a_{129}$
- (60) $a_9 a_{14} a_{62} a_{126} + a_{17} a_{65} a_{129}$
- (61) $a_9 a_{14} a_{126}^2 + a_{17} a_{129}^2 + a_{65} a_{81} a_{129}$
- (62) $a_9 a_{14} a_{193}$
- (63) $a_9 a_{17}^2$
- (64) $a_9 a_{17} a_{33} + a_{14}^3 a_{17}$
- (65) $a_9 a_{17} a_{65} + a_{14}^3 a_{49} + a_{14} a_{17} a_{30}^2$
- (66) $a_9 a_{17} a_{126}^3 + a_{17} a_{129}^3$
- (67) $a_9 a_{17} a_{129} + a_{14} a_{30} a_{49} a_{62} + a_{30}^3 a_{65}$
- (68) $a_9 a_{17} a_{193}$
- (69) $a_9 a_{30}^2$
- (70) $a_9 a_{30} a_{33}$
- (71) $a_9 a_{30} a_{62} a_{126} + a_{33} a_{65} a_{129}$
- (72) $a_9 a_{30} a_{65} + a_9 a_{33} a_{62}$
- (73) $a_9 a_{30} a_{126}^2 + a_{33} a_{129}^2 + a_{65} a_{81} a_{145}$
- (74) $a_9 a_{30} a_{129} + a_9 a_{33} a_{126}$
- (75) $a_9 a_{30} a_{193}$
- (76) $a_9 a_{33}^2$
- (77) $a_9 a_{33} a_{65} + a_{14}^2 a_{30} a_{49} + a_{17} a_{30}^3$
- (78) $a_9 a_{33} a_{126}^3 + a_{33} a_{129}^3$
- (79) $a_9 a_{33} a_{129} + a_{17} a_{30} a_{62}^2 + a_{30}^3 a_{81}$
- (80) $a_9 a_{33} a_{193}$
- (81) $a_9 a_{49}$
- (82) $a_9 a_{62}^2$

- (83) $a_9 a_{62} a_{65}$
- (84) $a_9 a_{62} a_{126}^2 + a_{65} a_{129}^2 + a_{81} a_{97} a_{145}$
- (85) $a_9 a_{62} a_{129} + a_9 a_{65} a_{126}$
- (86) $a_9 a_{62} a_{193}$
- (87) $a_9 a_{65}^2$
- (88) $a_9 a_{65} a_{126}^3 + a_{65} a_{129}^3$
- (89) $a_9 a_{65} a_{129} + a_{17} a_{62}^3 + a_{30}^2 a_{62} a_{81}$
- (90) $a_9 a_{65} a_{193}$
- (91) $a_9 a_{81}$
- (92) $a_9 a_{97}$
- (93) $a_9 a_{126} a_{193}$
- (94) $a_9 a_{129}^2 + a_{17} a_{62}^2 a_{126} + a_{30}^2 a_{81} a_{126}$
- (95) $a_9 a_{129} a_{193}$
- (96) $a_9 a_{145}$
- (97) $a_9 a_{161}$
- (98) $a_{14}^7 + a_{49}^2$
- (99) $a_{14}^6 a_{30} + a_{49} a_{65}$
- (100) $a_{14}^6 a_{62} + a_{14}^4 a_{30}^3 + a_{65} a_{81}$
- (101) $a_{14}^5 a_{30}^2 + a_{49} a_{81}$
- (102) $a_{14}^5 a_{30} a_{62} a_{193} + a_{17} a_{30}^5 a_{62} a_{126} + a_{33} a_{129} a_{193}$
- (103) $a_{14}^5 a_{62}^2 + a_{49} a_{145}$
- (104) $a_{14}^4 a_{17}$
- (105) $a_{14}^4 a_{30}^3 a_{193} + a_{17} a_{129} a_{193} + a_{30}^4 a_{33} a_{62}^3 + a_{65} a_{81} a_{193} + a_{81} a_{97} a_{161}$
- (106) $a_{14}^4 a_{30}^2 a_{62} + a_{49} a_{129}$
- (107) $a_{14}^4 a_{49}$
- (108) $a_{14}^3 a_{17}^2$

- (109) $a_{14}^3 a_{30}^4 + a_{81}^2$
- (110) $a_{14}^3 a_{30}^3 a_{62}^2 + a_{49} a_{62} a_{145} + a_{49} a_{81} a_{126} + a_{62} a_{65} a_{129} + a_{62} a_{97}^2 + a_{65}^2 a_{126}$
- (111) $a_{14}^3 a_{30}^3 a_{62} a_{193} + a_{30}^6 a_{81} a_{126} + a_{49} a_{145} a_{193} + a_{65} a_{129} a_{193} + a_{97}^2 a_{193}$
- (112) $a_{14}^2 a_{17} a_{30}$
- (113) $a_{14}^2 a_{30}^5 + a_{49} a_{129} + a_{81} a_{97}$
- (114) $a_{14}^2 a_{30}^2 a_{49} + a_{17} a_{30}^4$
- (115) $a_{14}^2 a_{49} a_{62} + a_{17} a_{30}^2 a_{62}$
- (116) $a_{14} a_{17}^2 a_{30}$
- (117) $a_{14} a_{17} a_{62} + a_{14} a_{30} a_{49} + a_{30}^2 a_{33}$
- (118) $a_{14} a_{17} a_{126} + a_{30}^2 a_{97} + a_{30} a_{62} a_{65}$
- (119) $a_{14} a_{17} a_{193} + a_{49}^2 a_{126} + a_{62} a_{81}^2$
- (120) $a_{14} a_{30}^6 + a_{49} a_{145} + a_{97}^2$
- (121) $a_{14} a_{30}^3 a_{49} a_{62} + a_{30}^5 a_{65}$
- (122) $a_{14} a_{30} a_{49} a_{62}^3 + a_{30}^3 a_{62}^2 a_{65}$
- (123) $a_{14} a_{33} + a_{17} a_{30}$
- (124) $a_{14} a_{49}^2$
- (125) $a_{14} a_{49} a_{126} + a_{30} a_{33} a_{126} + a_{30} a_{62} a_{97} + a_{62}^2 a_{65}$
- (126) $a_{14} a_{49} a_{193} + a_{30} a_{33} a_{193} + a_{62} a_{97}^2$
- (127) $a_{14} a_{65} + a_{17} a_{62} + a_{30} a_{49}$
- (128) $a_{14} a_{81} + a_{30} a_{65} + a_{33} a_{62}$
- (129) $a_{14} a_{97} + a_{30} a_{81} + a_{49} a_{62}$
- (130) $a_{14} a_{129} + a_{17} a_{126} + a_{62} a_{81}$
- (131) $a_{14} a_{145} + a_{30} a_{129} + a_{33} a_{126} + a_{62} a_{97}$
- (132) $a_{14} a_{161} + a_{30} a_{145} + a_{49} a_{126}$
- (133) a_{17}^3
- (134) $a_{17}^2 a_{30}^4$

- (135) $a_{17}^2 a_{33}$
- (136) $a_{17}^2 a_{62} + a_{30} a_{33}^2$
- (137) $a_{17}^2 a_{65}$
- (138) $a_{17}^2 a_{126} + a_{30} a_{65}^2 + a_{49}^2 a_{62}$
- (139) $a_{17}^2 a_{129}$
- (140) $a_{17}^2 a_{193}$
- (141) $a_{17} a_{30}^6$
- (142) $a_{17} a_{30}^5 a_{193} + a_{30}^2 a_{49} a_{62}^3 a_{65}$
- (143) $a_{17} a_{30}^3 a_{33}$
- (144) $a_{17} a_{30}^2 a_{62}^2 + a_{30}^4 a_{81}$
- (145) $a_{17} a_{30} a_{33} a_{62}$
- (146) $a_{17} a_{30} a_{33} a_{126} + a_{17} a_{62}^2 a_{65} + a_{30} a_{49} a_{62} a_{65}$
- (147) $a_{17} a_{30} a_{62}^3 a_{126}^2 + a_{30}^3 a_{62} a_{81} a_{126}^2 + a_{33} a_{65} a_{129}^3$
- (148) $a_{17} a_{30} a_{62}^2 a_{126}^3 + a_{30}^3 a_{81} a_{126}^3 + a_{33} a_{129}^4$
- (149) $a_{17} a_{30} a_{65} + a_{17} a_{33} a_{62}$
- (150) $a_{17} a_{30} a_{129} + a_{17} a_{33} a_{126} + a_{49} a_{62} a_{65}$
- (151) $a_{17} a_{33}^2$
- (152) $a_{17} a_{33} a_{62}^2 a_{126}^4 + a_{30}^2 a_{49} a_{65} a_{126}^4 + a_{33} a_{129}^5$
- (153) $a_{17} a_{33} a_{62} a_{65}$
- (154) $a_{17} a_{33} a_{65} a_{129}^2 + a_{30}^4 a_{62} a_{65} a_{126} + a_{30}^3 a_{33} a_{62}^2 a_{126} + a_{30}^3 a_{62}^3 a_{97} + a_{30}^2 a_{62}^4 a_{65}$
- (155) $a_{17} a_{33} a_{129}^3 + a_{30}^4 a_{65} a_{126}^2 + a_{30}^3 a_{33} a_{62} a_{126}^2 + a_{30}^3 a_{62}^2 a_{97} a_{126} + a_{30}^2 a_{62}^3 a_{65} a_{126}$
- (156) $a_{17} a_{33} a_{193} + a_{49} a_{65} a_{129}$
- (157) $a_{17} a_{49}$
- (158) $a_{17} a_{62}^4 + a_{30}^2 a_{62}^2 a_{81}$
- (159) $a_{17} a_{62}^3 a_{65} + a_{30} a_{49} a_{62}^2 a_{65}$
- (160) $a_{17} a_{62}^3 a_{126}^3 + a_{30}^2 a_{62} a_{81} a_{126}^3 + a_{65} a_{129}^4$

- (161) $a_{17}a_{62}^2a_{65}a_{126}^4 + a_{30}a_{49}a_{62}a_{65}a_{126}^4 + a_{65}a_{129}^5$
- (162) $a_{17}a_{62}^2a_{193} + a_{30}^2a_{81}a_{193}$
- (163) $a_{17}a_{62}a_{129} + a_{17}a_{65}a_{126} + a_{30}a_{49}a_{129} + a_{62}a_{65}a_{81}$
- (164) $a_{17}a_{65}^2$
- (165) $a_{17}a_{65}a_{129}^3 + a_{30}^3a_{62}a_{65}a_{126}^2 + a_{30}^2a_{33}a_{62}^2a_{126}^2 + a_{30}^2a_{62}^3a_{97}a_{126} + a_{30}a_{62}^4a_{65}a_{126}$
- (166) $a_{17}a_{65}a_{193} + a_{65}a_{81}a_{129}$
- (167) $a_{17}a_{81} + a_{49}^2$
- (168) $a_{17}a_{97} + a_{49}a_{65}$
- (169) $a_{17}a_{129}^4 + a_{30}^3a_{65}a_{126}^3 + a_{30}^2a_{33}a_{62}a_{126}^3 + a_{30}^2a_{62}^2a_{97}a_{126}^2 + a_{30}a_{62}^3a_{65}a_{126}^2$
- (170) $a_{17}a_{129}^2a_{193} + a_{81}a_{97}a_{145}^2$
- (171) $a_{17}a_{145} + a_{81}^2$
- (172) $a_{17}a_{161} + a_{49}a_{129} + a_{81}a_{97}$
- (173) $a_{30}^7a_{33} + a_{49}a_{65}a_{129}$
- (174) $a_{30}^7a_{81} + a_{65}a_{81}a_{145}$
- (175) $a_{30}^7a_{97} + a_{30}^5a_{33}a_{62}^2 + a_{81}a_{97}a_{129}$
- (176) $a_{30}^6a_{33}a_{62} + a_{65}a_{81}a_{129}$
- (177) $a_{30}^6a_{62}a_{81} + a_{81}a_{97}a_{145}$
- (178) $a_{30}^6a_{62}a_{97} + a_{49}a_{145}^2 + a_{81}a_{97}a_{161}$
- (179) $a_{30}^6a_{65}$
- (180) $a_{30}^5a_{33}a_{62}^3 + a_{49}a_{65}a_{126}a_{129} + a_{62}a_{81}a_{97}a_{129}$
- (181) $a_{30}^5a_{33}a_{126} + a_{30}^5a_{62}a_{97} + a_{30}^4a_{62}^2a_{65}$
- (182) $a_{30}^5a_{33}a_{193} + a_{30}^3a_{49}^2a_{62}a_{126} + a_{30}a_{49}^2a_{62}^4$
- (183) $a_{30}^5a_{62}^2a_{65} + a_{30}^4a_{33}a_{62}^3 + a_{49}a_{145}^2$
- (184) $a_{30}^5a_{62}^2a_{81} + a_{65}a_{129}a_{161}$
- (185) $a_{30}^5a_{65}a_{126} + a_{30}^4a_{33}a_{62}a_{126} + a_{30}^4a_{62}^2a_{97} + a_{30}^3a_{62}^3a_{65}$
- (186) $a_{30}^4a_{33}a_{62}^4 + a_{62}a_{81}a_{97}a_{161} + a_{65}a_{81}a_{126}a_{129}$

- (187) $a_{30}^4 a_{33} a_{62}^3 a_{193} + a_{49}^2 a_{62}^7 + a_{49} a_{145}^2 a_{193} + a_{81} a_{97} a_{161} a_{193}$
- (188) $a_{30}^4 a_{49}$
- (189) $a_{30}^4 a_{62}^4 a_{97} + a_{30}^3 a_{62}^5 a_{65} + a_{49} a_{126} a_{145}^2$
- (190) $a_{30}^3 a_{33}^2$
- (191) $a_{30}^3 a_{49} a_{65}$
- (192) $a_{30}^3 a_{62}^2 a_{65} a_{126} + a_{30}^2 a_{33} a_{62}^3 a_{126} + a_{30}^2 a_{62}^4 a_{97} + a_{30} a_{62}^5 a_{65}$
- (193) $a_{30}^3 a_{65} a_{193} + a_{30}^2 a_{33} a_{62} a_{193} + a_{49}^2 a_{62}^2 a_{126} + a_{62}^3 a_{81}^2$
- (194) $a_{30}^3 a_{129} + a_{30}^2 a_{33} a_{126} + a_{30}^2 a_{62} a_{97} + a_{30} a_{62}^2 a_{65} + a_{33} a_{62}^3$
- (195) $a_{30}^2 a_{33}^2 a_{62}^5 + a_{81} a_{97} a_{129}^2$
- (196) $a_{30}^2 a_{49}^2 a_{62}^2 + a_{30} a_{33}^2 a_{62}^3$
- (197) $a_{30}^2 a_{49} a_{62}^5 a_{65} + a_{81} a_{97} a_{145} a_{161}$
- (198) $a_{30}^2 a_{49} a_{129}$
- (199) $a_{30}^2 a_{65}^2 + a_{33}^2 a_{62}^2$
- (200) $a_{30}^2 a_{145} + a_{62}^2 a_{81}$
- (201) $a_{30} a_{33}^2 a_{62}^6 + a_{81} a_{97} a_{145}^2$
- (202) $a_{30} a_{33}^2 a_{126} + a_{30} a_{62} a_{65}^2 + a_{49}^2 a_{62}^2$
- (203) $a_{30} a_{33} a_{65} + a_{30} a_{49}^2 + a_{33}^2 a_{62}$
- (204) $a_{30} a_{33} a_{129} + a_{49} a_{62} a_{81} + a_{62} a_{65}^2$
- (205) $a_{30} a_{49}^2 a_{62}^6 + a_{81} a_{97} a_{161}^2$
- (206) $a_{30} a_{49} a_{62}^6 a_{65} + a_{65} a_{129} a_{161}^2$
- (207) $a_{30} a_{49} a_{65} a_{193} + a_{62} a_{65} a_{81} a_{129}$
- (208) $a_{30} a_{49} a_{81} + a_{30} a_{65}^2 + a_{33} a_{62} a_{65} + a_{49}^2 a_{62}$
- (209) $a_{30} a_{49} a_{145} + a_{33} a_{62} a_{129} + a_{33} a_{65} a_{126} + a_{49}^2 a_{126} + a_{62} a_{81}^2$
- (210) $a_{30} a_{62}^6 a_{65}^2 + a_{49} a_{145}^2 a_{193}$
- (211) $a_{30} a_{65} a_{81} + a_{49} a_{62} a_{65}$
- (212) $a_{30} a_{65} a_{129} + a_{33} a_{62} a_{129} + a_{49}^2 a_{126} + a_{62} a_{81}^2$

- (213) $a_{30}a_{65}a_{145} + a_{62}a_{81}a_{97}$
- (214) $a_{30}a_{81}^2 + a_{33}^2a_{126} + a_{62}a_{65}^2$
- (215) $a_{30}a_{81}a_{97} + a_{62}a_{65}a_{81}$
- (216) $a_{30}a_{81}a_{129} + a_{49}a_{62}a_{129} + a_{49}a_{65}a_{126} + a_{62}a_{81}a_{97}$
- (217) $a_{30}a_{81}a_{145} + a_{49}a_{62}a_{145} + a_{49}a_{81}a_{126} + a_{62}a_{65}a_{129} + a_{62}a_{97}^2 + a_{65}^2a_{126}$
- (218) $a_{30}a_{97}^2 + a_{49}^2a_{126} + a_{62}a_{81}^2$
- (219) $a_{30}a_{97}a_{129} + a_{62}a_{65}a_{129} + a_{65}^2a_{126}$
- (220) $a_{30}a_{97}a_{145} + a_{62}a_{65}a_{145}$
- (221) $a_{30}a_{97}a_{193} + a_{62}a_{65}a_{193} + a_{97}^2a_{126}$
- (222) $a_{30}a_{129}^2 + a_{33}a_{126}a_{129} + a_{62}a_{81}a_{145} + a_{62}a_{97}a_{129} + a_{81}^2a_{126}$
- (223) $a_{30}a_{129}a_{145} + a_{62}a_{81}a_{161} + a_{81}a_{97}a_{126}$
- (224) $a_{30}a_{145}^2 + a_{62}a_{97}a_{161} + a_{62}a_{129}^2 + a_{65}a_{126}a_{129} + a_{97}^2a_{126}$
- (225) $a_{30}a_{161} + a_{62}a_{129} + a_{65}a_{126}$
- (226) a_{33}^3
- (227) $a_{33}^2a_{62}^8 + a_{62}a_{97}a_{129}^2a_{145} + a_{81}a_{97}a_{126}a_{129}^2$
- (228) $a_{33}^2a_{65}$
- (229) $a_{33}^2a_{129}$
- (230) $a_{33}^2a_{193}$
- (231) $a_{33}a_{49}$
- (232) $a_{33}a_{62}a_{65}a_{129}$
- (233) $a_{33}a_{62}a_{129}^2 + a_{33}a_{65}a_{126}a_{129} + a_{62}a_{65}a_{81}a_{145}$
- (234) $a_{33}a_{65}^2$
- (235) $a_{33}a_{65}a_{129}^5$
- (236) $a_{33}a_{65}a_{193} + a_{65}a_{81}a_{145}$
- (237) $a_{33}a_{81} + a_{49}a_{65}$
- (238) $a_{33}a_{97} + a_{49}a_{81}$

- (239) $a_{33}a_{129}^6$
- (240) $a_{33}a_{129}^2a_{193} + a_{81}a_{97}a_{145}a_{161}$
- (241) $a_{33}a_{145} + a_{49}a_{129} + a_{81}a_{97}$
- (242) $a_{33}a_{161} + a_{49}a_{145} + a_{97}^2$
- (243) a_{49}^3
- (244) $a_{49}^2a_{62}^8 + a_{62}a_{81}a_{129}^2a_{193} + a_{81}a_{97}a_{126}a_{145}^2$
- (245) $a_{49}^2a_{65}$
- (246) $a_{49}^2a_{81}$
- (247) $a_{49}^2a_{129}$
- (248) $a_{49}^2a_{145}$
- (249) $a_{49}^2a_{161}$
- (250) $a_{49}^2a_{193}$
- (251) $a_{49}a_{62}^7a_{65}a_{129} + a_{81}a_{97}a_{145}a_{161}a_{193}$
- (252) $a_{49}a_{62}a_{145}^2 + a_{62}a_{81}a_{129}^2$
- (253) $a_{49}a_{62}a_{161} + a_{62}a_{65}a_{145} + a_{62}a_{81}a_{129}$
- (254) $a_{49}a_{65}^2$
- (255) $a_{49}a_{65}a_{81}$
- (256) $a_{49}a_{65}a_{129}a_{193} + a_{81}a_{97}a_{129}^2$
- (257) $a_{49}a_{65}a_{145}$
- (258) $a_{49}a_{65}a_{161} + a_{65}a_{81}a_{129}$
- (259) $a_{49}a_{81}^2$
- (260) $a_{49}a_{81}a_{129}$
- (261) $a_{49}a_{81}a_{145} + a_{65}a_{81}a_{129}$
- (262) $a_{49}a_{81}a_{161} + a_{65}a_{81}a_{145}$
- (263) $a_{49}a_{81}a_{193} + a_{81}a_{97}a_{145}$
- (264) $a_{49}a_{97} + a_{65}a_{81}$

- (265) $a_{49}a_{129}^2$
- (266) $a_{49}a_{129}a_{145} + a_{81}a_{97}a_{145}$
- (267) $a_{49}a_{129}a_{161} + a_{49}a_{145}^2 + a_{81}a_{97}a_{161}$
- (268) $a_{49}a_{145}^3$
- (269) $a_{49}a_{145}^2a_{193}$
- (270) $a_{49}a_{145}a_{161} + a_{65}a_{129}a_{161}$
- (271) $a_{62}^7a_{65}a_{81}a_{129} + a_{65}a_{129}a_{161}^2a_{193}$
- (272) $a_{62}^7a_{81}a_{97}a_{129}^2 + a_{81}a_{97}a_{145}a_{161}a_{193}^2$
- (273) $a_{62}^7a_{81}a_{129}^3 + a_{65}a_{129}a_{161}^2a_{193}^2$
- (274) $a_{62}a_{65}a_{129}^2 + a_{62}a_{81}a_{97}a_{145}$
- (275) $a_{62}a_{65}a_{145}^2 + a_{62}a_{97}a_{129}^2$
- (276) $a_{62}a_{65}a_{161} + a_{62}a_{97}a_{129}$
- (277) $a_{62}a_{81}a_{97}a_{145}^2 + a_{62}a_{81}a_{129}^3$
- (278) $a_{62}a_{81}a_{129}^4$
- (279) $a_{62}a_{81}a_{129}^3a_{193}$
- (280) $a_{62}a_{81}a_{129}^2a_{193}^2$
- (281) $a_{62}a_{81}a_{129}a_{145}$
- (282) $a_{62}a_{81}a_{129}a_{161} + a_{62}a_{81}a_{145}^2 + a_{62}a_{97}a_{129}a_{145}$
- (283) $a_{62}a_{81}a_{145}^3 + a_{62}a_{129}^4 + a_{65}a_{126}a_{129}^3$
- (284) $a_{62}a_{81}a_{145}a_{161} + a_{62}a_{97}a_{129}a_{161} + a_{62}a_{129}^3 + a_{65}a_{126}a_{129}^2 + a_{81}a_{97}a_{126}a_{145}$
- (285) $a_{62}a_{81}a_{161}^2 + a_{62}a_{129}^2a_{145}$
- (286) $a_{62}a_{97}a_{129}^4$
- (287) $a_{62}a_{97}a_{129}^3a_{145}$
- (288) $a_{62}a_{97}a_{129}^3a_{193} + a_{62}a_{129}^4a_{161}$
- (289) $a_{62}a_{97}a_{129}^2a_{145}a_{193}$
- (290) $a_{62}a_{97}a_{129}^2a_{161} + a_{62}a_{129}^4 + a_{65}a_{126}a_{129}^3$

- (291) $a_{62}a_{97}a_{129}a_{145}^2 + a_{62}a_{129}^4 + a_{65}a_{126}a_{129}^3$
- (292) $a_{62}a_{97}a_{161}^2 + a_{62}a_{129}^2a_{161} + a_{62}a_{129}a_{145}^2 + a_{65}a_{126}a_{129}a_{161} + a_{65}a_{126}a_{145}^2$
- (293) $a_{62}a_{129}^5 + a_{65}a_{126}a_{129}^4$
- (294) $a_{62}a_{129}^4a_{145}$
- (295) $a_{62}a_{129}^4a_{161}^2$
- (296) $a_{62}a_{129}^4a_{161}a_{193}^2$
- (297) $a_{62}a_{129}^4a_{193}^3$
- (298) $a_{62}a_{129}^3a_{145}^2$
- (299) $a_{62}a_{129}^2a_{145}^4$
- (300) a_{65}^3
- (301) $a_{65}^2a_{81}$
- (302) $a_{65}^2a_{129}$
- (303) $a_{65}^2a_{145}$
- (304) $a_{65}^2a_{161}$
- (305) $a_{65}^2a_{193}$
- (306) $a_{65}a_{81}^2$
- (307) $a_{65}a_{81}a_{129}^2$
- (308) $a_{65}a_{81}a_{129}a_{193} + a_{81}a_{97}a_{145}^2$
- (309) $a_{65}a_{81}a_{145}^2 + a_{81}a_{97}a_{129}^2$
- (310) $a_{65}a_{81}a_{145}a_{193} + a_{81}a_{97}a_{145}a_{161}$
- (311) $a_{65}a_{81}a_{161} + a_{81}a_{97}a_{129}$
- (312) $a_{65}a_{97} + a_{81}^2$
- (313) $a_{65}a_{129}^6$
- (314) $a_{65}a_{129}^2a_{161} + a_{81}a_{97}a_{145}a_{161}$
- (315) $a_{65}a_{129}^2a_{193} + a_{81}a_{97}a_{145}a_{193}$
- (316) $a_{65}a_{129}a_{145} + a_{81}a_{97}a_{161}$

$$(317) \quad a_{65}a_{129}a_{161}^3 + a_{81}a_{97}a_{145}a_{161}a_{193}$$

$$(318) \quad a_{65}a_{129}a_{161}^2a_{193}^3$$

$$(319) \quad a_{81}^3$$

$$(320) \quad a_{81}^2a_{97}$$

$$(321) \quad a_{81}^2a_{129}$$

$$(322) \quad a_{81}^2a_{145}$$

$$(323) \quad a_{81}^2a_{161}$$

$$(324) \quad a_{81}^2a_{193}$$

$$(325) \quad a_{81}a_{97}^2$$

$$(326) \quad a_{81}a_{97}a_{129}^3$$

$$(327) \quad a_{81}a_{97}a_{129}a_{145}$$

$$(328) \quad a_{81}a_{97}a_{129}a_{161} + a_{81}a_{97}a_{145}^2$$

$$(329) \quad a_{81}a_{97}a_{129}a_{193} + a_{81}a_{97}a_{161}^2$$

$$(330) \quad a_{81}a_{97}a_{145}^3$$

$$(331) \quad a_{81}a_{97}a_{145}^2a_{161}$$

$$(332) \quad a_{81}a_{97}a_{145}^2a_{193}$$

$$(333) \quad a_{81}a_{97}a_{145}a_{161}^2$$

$$(334) \quad a_{81}a_{97}a_{145}a_{161}a_{193}^3$$

$$(335) \quad a_{81}a_{97}a_{161}^3$$

$$(336) \quad a_{81}a_{97}a_{161}^2a_{193}$$

$$(337) \quad a_{97}^3$$

$$(338) \quad a_{97}^2a_{129}$$

$$(339) \quad a_{97}^2a_{145}$$

$$(340) \quad a_{97}^2a_{161}$$

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